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FOLIATIONS WITH A KUPKA COMPONENT OF CODIMENSION GREATER THAN ONE.

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ABSTRACT. We consider holomorphic foliations of complex codimension ≥ 2 with a compact connected component of the Kupka set.

1. SINGULAR HOLOMORPHIC FOLIATIONS

The set of codimension one holomorphic foliations with singularities on compact complex manifolds, is in a natural way a complex projective variety. In some cases, the structure of the singular set determines the global behavior of the foliation, that is the case for codimension one holomorphic foliations whose singular set, is a compact connected component of the Kupka (Definition (1.2)) set and such that, the normal bundle of the foliation extends to a very ample holomorphic line bundle.

Let $U \subset \mathbb{C}^{n+p}$ $n \geq 2$ be an open set. We say that a holomorphic p–form Ω in U, with $1 \leq p$, is **integrable**, if there exists an open covering $\{U_\alpha\}$ of $U - S_\Omega$, where $S_\Omega = \{z \in U \mid \Omega(z) = 0\}$, such that

- $\Omega|_{U_\alpha} = \eta_1 \wedge \cdots \wedge \eta_p$.
- $d\eta_i = \sum \omega_{ij} \wedge \eta_j$.

The first condition implies that the sheaf $\Theta_{\mathcal{F}} = \{\mathbf{X} \in \Theta \mid \imath_{\mathbf{X}}\Omega = 0\}$ outside the singular set S_Ω, is locally free and defines an n–plane field of $\mathbb{C}^{n+p}$ and the second implies that, for each $z \in U - S_\Omega$, there exists a codimension p analytic submanifold $\mathcal{L}_z$ whose tangent space $T_z\mathcal{L}$ at z is generated by $\{\mathbf{X}(z) \mid \mathbf{X} \in \Theta_{\mathcal{F}}\}$.

Now, let M be a complex manifold of complex dimension $n+p$, a codimension p holomorphic foliation with singularities on M may be given by the following data:

- An open covering $\mathfrak{U} = \{U_\alpha\}$ of M.
- A family of integrable p–forms Ω_α, defined on the open set U_α which satisfies the **cocycle condition**, that is
$$\Omega_\alpha = \lambda_{\alpha\beta} \cdot \Omega_\beta \quad \text{whenever} \quad U_{\alpha\beta} = U_\alpha \cap U_\beta \neq \emptyset$$
where $\lambda_{\alpha\beta} \in H^1(\mathfrak{U}, \mathcal{O}^*)$.

If we denote by L the holomorphic line bundle defined by the cocycle $\{\lambda_{\alpha\beta}\}$, these p–forms Ω_α, glue in order to obtain a holomorphic section of the vector bundle $\wedge^p T^*M \otimes L$.

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Two integrable sections Ω_1 and Ω_2 of the bundle $\wedge^p T^*M \otimes L$ are equivalent, if there exits a never vanishing holomorphic function $\lambda : M \to \mathbb{C}^*$ such that $\Omega_1 = \lambda \cdot \Omega_2$, it is clear that both p–forms define the same foliation.

In what follows, we are going to denote by $\Omega^p(L)$ the sheaf of holomorphic sections of the vector bundle $\wedge^p T^*M \otimes L$. In the case of the projective space, a line bundle is determined by an integer c, its Chern class, which is denoted by $\mathcal{O}(c)$ and the sheaf of holomorphic sections of $\wedge^p T^*\mathbb{P}^N \otimes \mathcal{O}(c)$ by $\Omega^p(c)$.

This motivates the following definition:

Definition 1.1. A Codimension p holomorphic foliation with singularities on a complex manifold M is an equivalence of integrable sections $\Omega \in H^0\big(M, \Omega^p(L)\big)$.

The singular set of the foliation represented by the section Ω, is defined by

$$S_\Omega = \{x \in M \mid \Omega(x) = 0\}.$$

The aim of this note, is to study some properties of the so called **Kupka singular set** for holomorphic foliations of codimension $p \geq 2$. These set was introduced by I. Kupka in [3] for integrable 1–forms and by A. Medeiros in [6] in the general case.

Definition 1.2. The **Kupka singular set** of an integrable p–form is defined by

$$K_\Omega := \{p \in M \mid \Omega(p) = 0, \quad d\Omega(p) \neq 0\}.$$

The following theorem, gives us the main property of the Kupka set [6].

Theorem 1.3. *Let $K \subset K_\Omega$ be a connected component, then there exists an open covering $\mathfrak{U} = \{U_\alpha\}$ of a neighborhood of K by charts of M, such that*

$$\begin{aligned} \phi_\alpha : U_\alpha &\to \mathbb{C}^{p+1} \times \mathbb{C}^{n-1} \\ u &\mapsto \big(x_\alpha(u), y_\alpha(u)\big), \end{aligned}$$

a holomorphic vector field, called the **transversal type**

$$\mathbf{X}(x,y) = \sum_{i=1}^{p+1} X_i(x) \frac{\partial}{\partial x^i},$$

such that the p–form $\Omega_\alpha = \phi_\alpha^* \imath_{\mathbf{X}} dx^1 \wedge \cdots \wedge dx^{p+1}$ represents the foliation on U_α.

Observe that in the coordinates (x_α, y_α) the foliation is represented by the holomorphic p–form of equation

$$\Omega_\alpha(x_\alpha^1, \ldots, x_\alpha^{p+1}) = \sum_{i=1}^{p+1} (-1)^i X_i(x_\alpha)\, dx_\alpha^1 \wedge \cdots \wedge \widehat{dx_\alpha^i} \wedge \cdots \wedge dx_\alpha^{p+1},$$

as usual, $\widehat{dx_\alpha^i}$ means that this term is missing,

An important consequence of this theorem, is that the tangent sheaf of the foliation $\Theta_{\mathcal{F}}$, is locally free in points of the Kupka set, namely, in the coordinate system $(x_\alpha, y_\alpha) = (x_\alpha^1, \dots, x_a^{p+1}, y_\alpha^1, \dots, y_\alpha^{n-1})$, it is generated by the vector fields

$$\Theta_{\mathcal{F}}(U_\alpha) = \left\{ \mathbf{X} = \sum_{i=1}^{p+1} X_i(x_\alpha) \frac{\partial}{\partial x_\alpha^i}, \frac{\partial}{\partial y_\alpha^1}, \dots, \frac{\partial}{\partial y_\alpha^{n-1}} \right\}.$$

We will say that the foliation has the **local product property**.

Remark 1.4. We have the following properties of the transversal type:

(1) The vector field $\mathbf{X}$ in Theorem (1.3), is that it is unique up multiplication by never vanishing holomorphic functions.
(2) The condition $d\Omega(0) \neq 0$ is equivalent to $Div\mathbf{X}(0) \neq 0$, in particular, the vector field $\mathbf{X}$ has always a non zero linear part $\mathbf{X}_1$, and it is unique up multiplication by non zero complex numbers.
(3) We say that the Kupka connected component is non degenerated if the linear $\mathbf{X}_1$ part of $\mathbf{X}$ has non zero eigenvalues.

Let M be a complex manifold of complex dimension $n+p$, and we always assume that $n \geq 2$. We are interested in codimension p foliations on M, whose singular set has a compact connected component of the Kupka set.

Let us give one example of a foliations with a compact Kupka set, observe how it is related with the Theorems (3.1) and (3.2).

Example 1. Consider the projective space $\mathbb{P}^{n+p}$, $\quad n \geq 2$, $p \geq 1$ and let $\{f_i\}_{i=0}^p$ homogeneous polynomials of degree d_i in $n+p+1$ variables. Let $n_0, \dots, n_p$ be relatively prime natural numbers such that $n_i d_i = n_j d_j$. We assume that $\{f_i = 0\}$ are smooth irreducible and reduced and $\{f_0 = \cdots = f_p = 0\}$ defines a smooth complete intersection.

The rational map $F : \mathbb{P}^{n+p} \to \mathbb{P}^p$ defined by $F(x) = [f_0^{n_0}(x) : \cdots : f_p^{n_p}]$ defines a codimension p foliation represented by a p–form $\Omega \in H^0(\mathbb{P}^{n+p}, \Omega^p(c))$, where the line bundle is $\mathcal{O}(c)$, where $c = d_0 + \cdots + d_p$, namely

$$\begin{aligned} \Omega &= (f_0 \cdots f_p) \sum_{i=0}^{p} (-1)^i \mu_i \frac{df_0}{f_0} \wedge \cdots \wedge \widehat{\frac{df_i}{f_i}} \wedge \cdots \frac{df_p}{f_p} \\ &= \sum_{i=0}^{p} (-1)^i \mu_i f_i df_0 \wedge \cdots \wedge \widehat{df_i} \wedge \cdots df_p \end{aligned}$$

where $\mu_i = n_0 \dots n_p / n_i$. The Kupka set is $K_\Omega = (f_0 = \cdots = f_p = 0)$, i. e. the base point of the map, the transversal type is the vector field

$$\mathbf{X} = \sum_{i=0}^{p} \mu_i x_i \frac{\partial}{\partial x_i},$$

and the normal bundle of the Kupka set $\nu_K(\mathbb{P}^{n+p}) = \bigoplus_{i=0}^{p} \mathcal{O}_K(d_i)$.

Since $\mu_i / \mu_j = n_j / n_i$, we have the equality

$$\mu_j c(\mathcal{O}_K(d_i)) - \mu_i c(\mathcal{O}_K(d_j)) = 0 \in H^2(K, \mathbb{Z}),$$

where $c(L)$ denotes the Chern class of a line bundle L (see equation (3.2)).

Recall that for $p=1$, the foliation is represented by the 1–form

$$\Omega = n_1 f_0 df_1 - n_0 f_1 df_0 \in H^0(\mathbb{P}^{n+1}, \Omega^1(d_0+d_1)),$$

moreover, if the Kupka set is a complete intersection, it is shown in [2] that the foliation must be of this type. It is natural to ask if the same is true for arbitrary $p>1$.

Observe that in the example above, the normal bundle $\nu_K(\mathbb{P}^{n+p})$ is the restriction of a rank $p+1$ holomorphic vector bundle $E \to \mathbb{P}^{n+p}$ with a section which defines the Kupka set. This property is always true for codimension one holomorphic foliations with a compact connected components of the Kupka set and with ample normal bundle, it is natural to ask if the same is true in the case of foliations of codimension $p \geq 2$.

2. Change of Coordinates

In this section, we are going to consider the codimension two case. Let

$$\mathbf{X}(x) = \sum_{i=1}^{3} X_i(x) \frac{\partial}{\partial x^i},$$

be the transversal vector field of a connected component K of the Kupka singular set of a codimension two holomorphic foliation on the complex manifold M of complex dimension $n+2$ with $n \geq 2$.

The foliation may be represented by the holomorphic 2–form

$$\begin{aligned} \Omega &= i_{\mathbf{X}} dx^1 \wedge dx^2 \wedge dx^3 \\ &= X_3\, dx^1 \wedge dx^2 + X_1\, dx^2 \wedge dx^3 + X_2\, dx^3 \wedge dx^1 \end{aligned}$$

Let $\{(U_\alpha, \phi_\alpha)\}$ be a family of charts of M, as in the theorem (1.3), so that

$$\begin{aligned} \phi_\alpha : U_\alpha &\to \mathbb{C}^3 \times \mathbb{C}^{n-1} \\ p &\mapsto \big(x_\alpha(p); y_\alpha(p)\big), \end{aligned}$$

and the holomorphic 2–form $\phi_\alpha^*(\Omega) := \Omega_\alpha$, represents the foliation on the open set U_α. We also have

$$\phi_\alpha^{-1}(0, y_\alpha) = K \cap U_\alpha$$

The change of coordinates

$$\phi_{\alpha\beta} = \phi_\alpha \circ \phi_\beta^{-1} = (\varphi_{\alpha\beta}, \psi_{\alpha\beta}) : \mathbb{C}^3 \times \mathbb{C}^{n-1} \to \mathbb{C}^3 \times \mathbb{C}^{n-1}$$

satisfies the relation

$$\phi_{\alpha\beta}^*(\Omega) = \Delta_{\alpha\beta} \cdot \Omega \quad \text{where} \quad \Delta_{\alpha\beta} \in \mathcal{O}^*\big(\phi_\alpha(U_\alpha \cap U_\beta)\big),$$

that implies the following systems of equations :

$$0 = X_3 \circ \phi_{\alpha\beta} \frac{\partial(\varphi^1_{\alpha\beta}, \varphi^2_{\alpha\beta})}{\partial(x^i_\beta, y^j_\beta)} + X_1 \circ \phi_{\alpha\beta} \frac{\partial(\varphi^2_{\alpha\beta}, \varphi^3_{\alpha\beta})}{\partial(x^i_\beta, y^j_\beta)} + X_2 \circ \phi_{\alpha\beta} \frac{\partial(\varphi^3_{\alpha\beta}, \varphi^1_{\alpha\beta})}{\partial(x^i_\beta, y^j_\beta)}$$

for all $i = 1,2,3$ and $j = 1, \ldots, n-1$.

$$0 = X_3 \circ \phi_{\alpha\beta} \frac{\partial(\varphi^1_{\alpha\beta}, \varphi^2_{\alpha\beta})}{\partial(y^i_\beta, y^j_\beta)} + X_1 \circ \phi_{\alpha\beta} \frac{\partial(\varphi^2_{\alpha\beta}, \varphi^3_{\alpha\beta})}{\partial(y^i_\beta, y^j_\beta)} + X_2 \circ \phi_{\alpha\beta} \frac{\partial(\varphi^3_{\alpha\beta}, \varphi^1_{\alpha\beta})}{\partial(y^i_\beta, y^j_\beta)}$$

for all $i, j = 1, \ldots, n-1$.

Now, we consider the expansion in power series

$$\varphi^i_{\alpha\beta}(x_\beta; y_\beta) := \sum_{\mathbf{I}} \varphi^{i,\mathbf{I}}_{\alpha\beta}(y_\beta) \cdot x^{\mathbf{I}}_\beta,$$

where $\varphi^{i,\mathbf{I}}_{\alpha\beta}$ are holomorphic functions of the variables $y_\beta = (y^1_\beta, \ldots, y^{n-1}_\beta)$, and $x_\beta = (x^1_\beta, x^2_\beta, x^3_\beta)$, and if $\mathbf{I} = (n_1, n_2, n_3)$, then $x^{\mathbf{I}}_\beta = (x^1_\beta)^{n_1}(x^2_\beta)^{n_3}(x^3_\beta)^{n_3}$.

We will denote by $\mathbf{j}$ the multi index $i_j = \delta_{ij}$, then the matrix

$$\Phi_{\alpha\beta} = \begin{pmatrix} \varphi^{\mathbf{11}}_{\alpha\beta}, & \varphi^{\mathbf{12}}_{\alpha\beta}, & \varphi^{\mathbf{13}}_{\alpha\beta} \\ \varphi^{\mathbf{21}}_{\alpha\beta}, & \varphi^{\mathbf{22}}_{\alpha\beta}, & \varphi^{\mathbf{23}}_{\alpha\beta} \\ \varphi^{\mathbf{31}}_{\alpha\beta}, & \varphi^{\mathbf{32}}_{\alpha\beta}, & \varphi^{\mathbf{33}}_{\alpha\beta} \end{pmatrix} \tag{2.1}$$

defines the cocycle of the normal bundle $\nu_K(M)$ of the Kupka set $K \subset M$.

3. Transversal Type with Diagonal Linear Part

In this section, we are going to applied the computation of section (2) when the linear part of the transversal field $\mathbf{X}_1$ is in diagonal form.

Consider a codimension two holomorphic foliation, with a compact connected component K with transversal type given by the holomorphic vector field on $\mathbb{C}^3$ defined by

$$\mathbf{X}(x) = \sum_{i=1}^{3} \left(\mu_i \cdot x^i + f_i(x)\right) \frac{\partial}{\partial x^i} \tag{3.1}$$

where $f_i(0) = \frac{\partial f_i}{\partial x^j}(0) = 0$, for all $i, j = 1, 2, 3$.

When $\mu_i \neq \mu_j$ for all $i, j = 1, 2, 3$, the matrix (2.1) is diagonal, so that, the normal bundle $\nu_K(M)$, splits in a direct sum of holomorphic line bundles $L_i := L_{\mu_i}$ each of them associated to the eigenvectors of the linear part $\mathbf{X}_1$ of $\mathbf{X}$.

Theorem 3.1. *Let K_0 be a compact connected component of the Kupka set with transversal type as (3.1)*

- *If $\mu_i \neq \mu_j$ for all $i, j \in \{1, 2, 3\}$, then the normal bundle splits in a direct sum of line bundles, $\nu_K(M) = L_1 \oplus L_2 \oplus L_3$.*
- *Assume that $\nu_K(M) = L_1 \oplus L_2 \oplus L_3$, then for all $i \neq j$ the following relation holds:*

$$\mu_i c(L_j) - \mu_j c(L_i) = 0 \in H^2(K, \mathbb{Z}).$$

The proof may be found in ([1]) and it is made by a direct calculation of the matrix (2.1). With the same technics, it may be generalized to the following theorem:

Theorem 3.2. *Let K_0 be a compact connected component of the Kupka set with transversal type given by the vector field*

$$X(x) = \sum_{i=0}^{p} \big(\mu_i \cdot x^i + f_i(x)\big) \frac{\partial}{\partial x^i} \quad \textit{where} \quad f_i(0) = \frac{\partial f_i}{\partial x^j}(0) = 0,$$

then

- *If $\mu_i \neq \mu_j$ for all $i, j \in \{0, \ldots, p\}$, then the normal bundle splits in a direct sum of line bundles associated to each proper subspace, $\nu_K(M) = L_0 \oplus \cdots \oplus L_p$.*
- *Assume that $\nu_K(M) = L_0 \cdots \oplus L_p$, then for all $i \neq j$ the following relation holds:*

$$\mu_i c(L_j) - \mu_j c(L_i) = 0 \in H^2(K, \mathbb{Z}). \tag{3.2}$$

As a first consequence of this result, also proved in ([1]), we have the following theorem, relating the local behavior of the foliation near the Kupka set K and the embedding $\jmath : K \hookrightarrow M$, which is measured with the topological properties of normal bundle of $K \subset M$.

Theorem 3.3. *Let K be a compact connected component of the Kupka set of a codimension 2 foliation with diagonal linear transversal type*

$$\mathbf{X}_1(x) = \sum_{i=1}^{3} \mu_i \cdot x^i \frac{\partial}{\partial x^i} \quad \textit{where} \quad \mu_i \neq \mu_j,$$

if the first Chern class $c_1(\nu_K(M)) \neq 0 \in H^2(K; \mathbb{C})$, then it is non degenerated and the numbers $\mu_i/\mu_j \in \mathbb{Q}$.

Proof. Observe that

- $\nu_K(M) = L_i \oplus L_2 \oplus L_3$ implies that
$$c_1(\nu_K(M)) = c(L_1) + c(L_2) + c(L_3),$$
then for some $i = 1, 2, 3$ we have $c(L_i) \neq 0$.
- The relation of the Chern classes given by the equation (3.2) implies that for $i = 1, 2, 3$ we have $\mu_i \neq 0$.
- Again, the equation (3.2) implies that $\mu_i/\mu_j \in \mathbb{Q}$

□

Remark 3.4. The condition $c_1(\nu_K(M)) \neq 0$ is natural: As in the codimension one case, we have the equality

$$\wedge^{p+1} \nu_K(M) = L|_K = \jmath^* L,$$

and by definition, we have $c_1(\nu_K(M)) = \jmath^* c(L)$ where $\jmath : K \to M$ denotes the inclusion. For example, if the line bundle L is sufficiently ample, that always is true in the projective space, and we may assume in the case of algebraic manifolds, we have that $c_1(\nu_K(M)) \neq 0$.

As a consequence of this last result, we have that the linear part of the transversal type is rigid under deformations of the foliation.

Corollary 3.5. *Let $\{\mathcal{F}_t | t \in \mathbb{C}^N\}$ be a family of codimension p holomorphic foliations such that $\mathcal{F}_0$ has a compact connected component K_0 with linear transversal type*

$$\mathbf{X_1} = \sum_{i=0}^{p} \mu_i x^i \frac{\partial}{\partial x^i}, \quad \mu_i \neq \mu_j, \quad i \neq j$$

If the first Chern class $c_1(\nu_K) \neq 0$ then any t sufficiently close to zero, $\mathcal{F}_t$ has a Kupka compact connected component K_t close to K_0 with the same linear transversal type.

Now, we consider the case where the linear part of the transversal type is radial:

$$\mathbf{X}(x) = \sum_{i=0}^{p} \left(x^i + f_i(x)\right) \cdot \frac{\partial}{\partial x^i}.$$

The transversal type is analytically linearizable, and we say that the foliation has **radial transversal type**. In this case, we are able to calculate the total Chern class of the normal bundle, thats only depends on the Chern class of the bundle L and the embedding $\jmath : K \hookrightarrow M$.

Theorem 3.6. *Let K be the Kupka, compact, connected component of a codimension p foliation $\mathcal{F}$ represented by a section of $H^0(M, \Omega^p(L))$. If K has radial transversal type, the normal bundle is projectively flat bundle, therefore the total Chern class of the normal bundle is*

$$c(\nu_K(M)) = \jmath^* \left(1 + \frac{c(L)}{p+1}\right)^{p+1} \in H^*(K, \mathbb{Z})$$

4. Transversal Type with non–semisimple Linear Part

In this section, we are going to applied the computation of section (2) when the linear part of the transversal field $\mathbf{X}_1$ is non semisimple, we are going to assume that the linear part of the transversal vector field is also non degenerated. Our main result of this section is the following Theorem.

Theorem 4.1. *Let K be a compact, connected component of a codimension two holomorphic foliation. If the transversal type has non–semisimple linear part, then the first Chern class of the normal bundle vanishes in $H^2(K, \mathbb{Z})$.*

Again, the idea of the proof of the therem (4.1), we must calculate the matrix (2.1) in two cases:

(1) The linear transversal type has the form

$$\mathbf{X}_1(x) = \begin{pmatrix} \mu & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

(2) The linear transversal type has the form

$$\mathbf{X}_1(x) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

4.1. **Case 1.** In this case, an explicit computation of the matrix (2.1) shows that there is an exact sequences of holomorphic vector bundles

$$\begin{aligned} 0 &\rightarrow L \rightarrow \nu_K(M) \rightarrow E \rightarrow 0 \\ 0 &\rightarrow L_1 \rightarrow E \rightarrow L_1 \rightarrow 0 \end{aligned}$$

Where L and L_1 are holomorphic line bundles, and E is a rank–two holomorphic vector bundle, then we have

$$c_1(\nu_K(M)) = c_1(L) + 2c_1(L_1).$$

On the other hand, we can prove the following relations.

$$\begin{aligned} c_1(L) &= \mu c_1(L_1) \\ c_1(L_1) &= 0, \end{aligned}$$

then $c_1(\nu_K(M)) = 0$.

4.2. **Case 2.** The divisor of equation $\{x^3 = 0\}$, is the only one smooth surface invariant by the foliation represented by the transversal vector field, therefore the preimages $\phi_\alpha^{-1}(x_\alpha^3 = 0)$ defines a smooth divisor D on a neighborhood U of the Kupka set which is saturated by leaves of the foliation. In this divisor, we have a codimension one holomorphic foliation with a Kupka phenomenon with non–semisimple linear part.

From the cocycles, we have exact sequences of holomorphic vector bundles

$$0 \rightarrow E \rightarrow \nu_K(M) \rightarrow L \rightarrow 0 \tag{4.1}$$

$$0 \rightarrow L \rightarrow E \rightarrow L \rightarrow 0 \tag{4.2}$$

moreover, the vector bundle $E = \nu_K(D)$, by a theorem in ([4]), we have that $c_1(E) = 0 \in H^2(K, \mathbb{Z})$, the exact sequence (4.2) implies that $c_1(E) = 2c_1(L) = 0$ and by the first exact sequence (4.1), we conclude that $c_1\big(\nu_K(M)\big) = c_1(E) + c_1(L)$ vanishes in $H^2(K, \mathbb{Z})$.

Finally, a combination of the theorems (4.1) and (3.1) we have.

Corollary 4.2. *Let $\mathcal{F}$ be a codimension two foliation with a compact, connected component $K_0 \subset K(\mathcal{F})$ with transversal type given by the radial vector field in $\mathbb{C}^3$. If the first Chern class of the normal bundle $c_1(\nu_K(M))$ does not vanish in $H^2(K, \mathbb{Z})$ then the transversal type is fixed under deformations of the foliation.*

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